Math Fact Fluency: Supporting the Learning and Retention of Basic Facts

SAC
January 19. 2024

Goals for Today

By the end of our meeting, we will be able to:

- Understand the Math goals of the School Improvement Plan (SIP)
- Gain a deeper understanding of the learning of basic facts
- Learn about some strategies and activities to help support your child's mastery of basic facts at home

Tobin Montessori School Improvement Plan

2023 to 2025

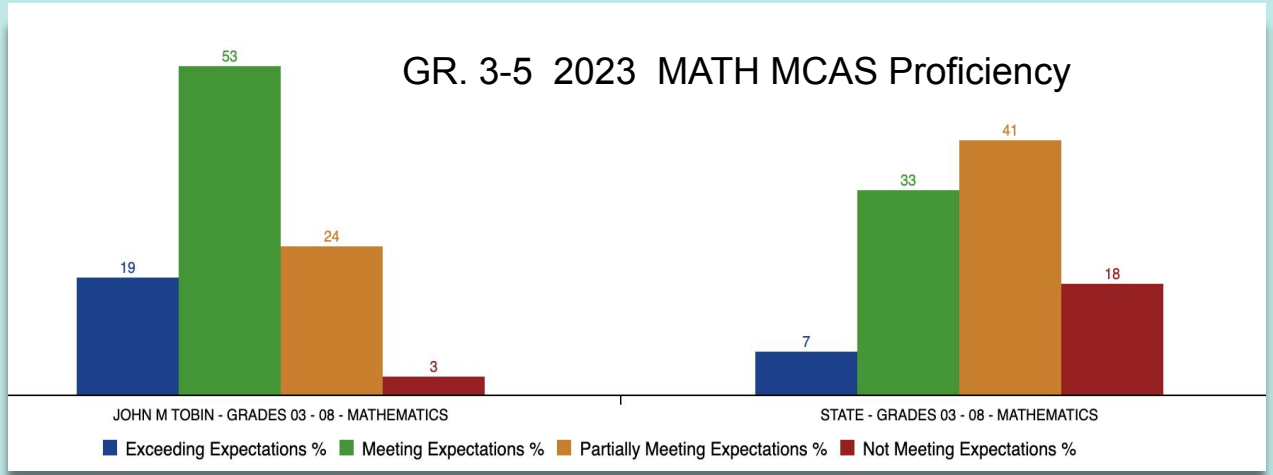
Math Goal:

By Spring 2025 all student demographic groups will meet or exceed the Massachusetts Accountability Targets on the MATH MCAS.

Students	Percentage Point Growth by 2025
All Students	3.0
Lowest Performing	10.2
High Needs	4.7
EL & Former EL	4.7
Students With Disabilities	4.4

Data Overview

Math



Tobin	State
19%	7%
53%	33%
24%	41%
3%	18%

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Key Successes

Math

Mathematics achievement - MCAS average composite scaled score - Non-high school							About the Data
Group	**2022 Achievement**	**2023 Achievement**	**Change**	**2023 Target**	**N**	**Points**	**Reason**
All Students	511.5	512.5	1.0	514.0	98	4	Recovery Path: Exceeded Target
Lowest Performing	496.8	496.7	-0.1	501.1	20	3	Path Forward: Met Target
High needs	503.6	505.8	2.2	505.8	46	3	Recovery Path: Met Target
Low income	498.1	502.5	4.4	500.4	26	4	Recovery Path: Exceeded Target
EL and Former EL	-	-	-	-	7	-	-
Students w/ disabilities	501.9	504.7	2.8	504.1	30	4	Recovery Path: Exceeded Target
Amer. Ind. or Alaska Nat.	-	-	-	-	-	-	-
Asian	-	-	-	-	12	-	-
Afr. Amer./Black	-	-	-	-	21	-	-
Hispanic/Latino	-	-	-	-	9	-	-
Multi-race, Non-Hisp./Lat.	-	-	-	-	11	-	-
Nat. Haw. or Pacif. Isl.	-	-	-	-	-	-	-
White	519.1	517.9	-1.2	520.8	45	4	Path Forward: Exceeded Target

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Key Areas of Need

Math

FACT Fluency!

	Gr. 3 Tobin	Gr. 3 State	+ /-	Gr. 4 Tobin	Gr. 4 State	+ /-	Gr. 5 Tobin	Gr. 5 State	+ /-
Operations and Algebraic Thinking	83%	59%	+24	75%	55%	+20	60%	47%	+13
Number and Operations in Base Ten	73%	53%	+20	78%	60%	+18	71%	55%	+16
Number and Operations Fractions	82%	58%	+24	72%	57%	+15	65%	51%	+14
Measurement and Data	75%	53%	+22	62%	46%	+16	61%	50%	+11
Geometry	76%	52%	+24	79%	66%	+13	65%	57%	+8

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Turn and Talk

What was your experience like when learning basic facts in elementary school?

Your Own Experience with Learning Basic Facts

- Speed drills
- Playing *Around the World* in class (the pressure!)
- Flashcards
- Timed tests
- Staying in for recess if you hadn’t memorized your time tables

Teachers were well-intentioned but...

... probably grew up with the belief that being good at math meant that:

- You had to be good at Memorization
- Speed was important

NCTM Research-based Position Statement on Procedural Fluency

“Basic facts should be taught using number relationships and reasoning strategies, not memorization. **Students who learn fact strategies outperform students who learn through other approaches**.”

WHY Focus on Using Strategies?

- Your child is much more likely to remember facts later on

- Your child is much less likely to have stress and anxiety

- A lack of automaticity with multiplication facts makes computing with fractions, simplifying algebraic expressions and many other higher-level mathematical skills significantly more difficult for students.

Our Math SIP: Key Area of Need to Address

- Basic facts are the foundation on which math computation is based.

- Our data shows that a lack of fluency with basic facts is hindering some students' progress in math, as inefficient ways of trying to solve math facts are **consuming their working memory,** thus **interfering with the ability/time to solve high cognitive demand problems.**

- Mastery of the basic facts for a student's grade level will increase their **confidence** and **self-worth as a mathematician.**

Our Work

- What do students need to know to be considered procedurally fluent?
- How do we make practice joyful and engaging?
- How do we assess fact fluency?

What is Procedural Fluency?

Computing with these 3 Components:

- Accuracy
- Efficiency
- Flexibility

Foundational Facts

- Just as concrete foundations must be poured before a house can be built, **some facts must be known to help solve for other facts.** Otherwise students are just left to simply memorize.

- **Ordering fact learning by ease of mastery** is more helpful for a **strategy-based approach.**

- For most students Foundational facts are easy to learn.

Addition Fact Fluency Learning Progression

Foundational Fact Sets

+/- 0, 1, 2

Doubles

Combos of 10

10 + ____

Derived Fact Strategies

Near Doubles

Making 10

Pretend-a-10

How do we develop Fact Fluency for +/- within 5?

Start with Foundational facts of +/- 0, 1, 2

- Students have the life experiences of “more”.
- Can I have one more?
- She has 2 more than me.

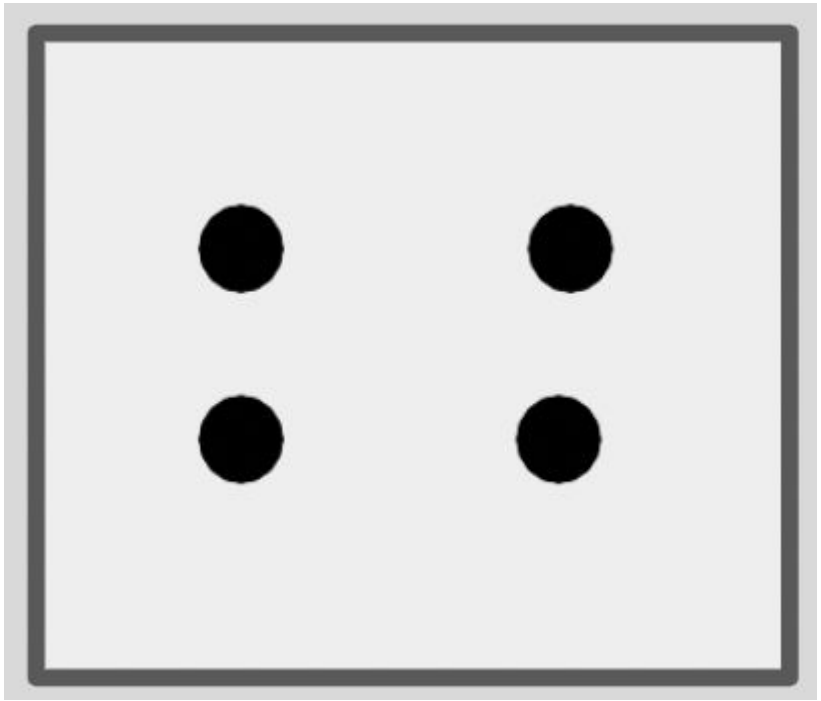
Early Number Concepts

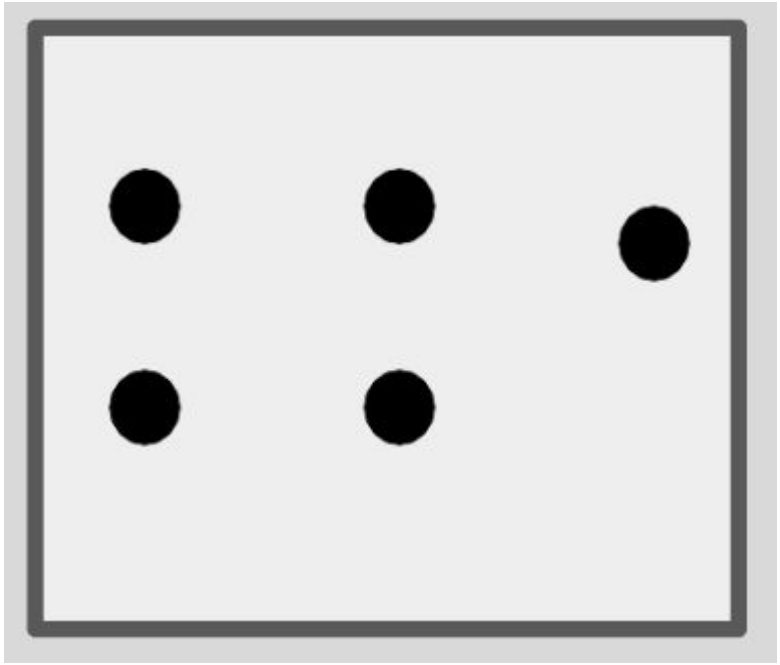
- Use lessons and Montessori materials to count and represent numbers. Be **intentional** about infusing the language of “one more” and “one less” into Monessori lessons

- Dot patterns cards

- Storytelling/Word Problems

Dot Cards





Early Understanding of Operations: Storytelling

- Use real life situations from the classroom

When working on **story problems**, the **key** is to have students use **materials, drawings, acting it out, or number line to represent and solve problems.**

Have students **write equations** to represent the story problem.

Multiplication Fact Fluency Learning Progression

Foundational Fact Sets

2s 10s 5s

1s 0s Squares

Derived Fact Strategies

Doubling: 4s, 8s Adding a Group: 3s, 6s Subtracting a Group: 9s

Break Apart Near Squares

FIGURE 4.8 Multiplication Table with Foundational Facts Highlighted

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

U. S. Convention of Representing Multiplication

4x3 is **4 groups of 3**, which is **3+3+3+3** or **4 rows by 3** columns.

Let's examine the order to teach the multiplication tables with strategies

2, 10, 5, 1, 0

The 2's Table

Students have had experiences with **adding doubles.**

So it makes sense to **relate 6 + 6** to **2 x 6**

Have students **model problems** involving 2's facts. Have students use materials and then **draw the representation** on paper with the **corresponding addition AND multiplication equations.**

Have students share solutions and **look for patterns** and then **generalize** that whenever they multiply by 2, they can think of the corresponding addition double.

Multiplication: 10s and 5s Table

10’s table is relatively quick because of students prior knowledge of place value.

With the 5s table, students may try to skip count here, but not efficient.

Have Students Discuss the Relationship Between Facts

To solve 5x 6

Think … 10 x 6 = 60

So 5 is half of 10, and 30 is half of 60

Therefore 5 x 6 = 30

Time for Practice!

Once strategies are established for the **Foundational Facts** of **2s, 10s, 5s, 1s and 0s, sufficient practice** is needed to **promote mastery** of these important facts.

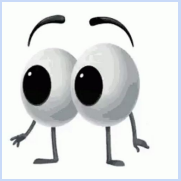
Derived Facts

Times Tables - 2s and 4s

Copy this table into your math book.

Continue the table through x10.

What do you notice?



Leave a column blank at the end for later.

X	2	4	
1	2	4	
2	4	8	
3	6	12	
4	8	16	
5	10	20	

Doubling Strategy for 4s

FIGURE 5.4 **Doubling Strategy Reasoning**

Mental Process in Symbols	What Students Might Think or Say as They Solve
$4 \times 6 = ?$	I know that 4 times a number is the same as multiplying the number by 2 twice.
$2 \times (2 \times 6) = ?$	I double 6 (multiply it by 2). I know that equals 12.
$2 \times 12 = 24$	I now double 12. I know that equals 24.

This is the same as writing:
2 x 6 = 12
2 x 6 = 12
12 + 12 = 24 or 2 x 12 = 24

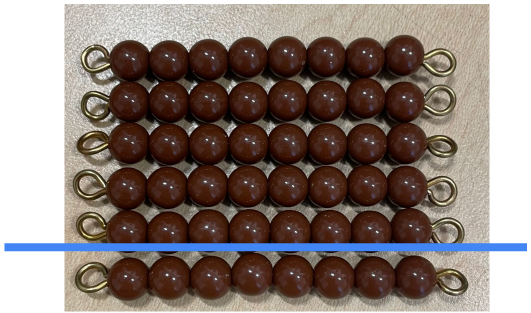
Adding a Group - adding a nearby known fact

What Student Might Think as they Solve

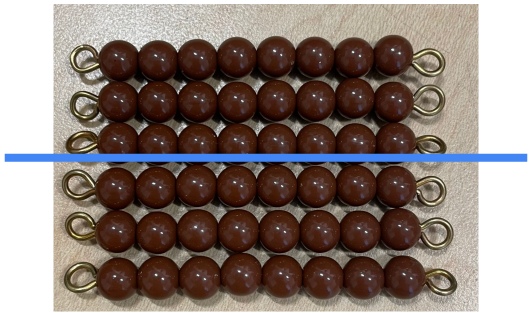
6 x 7 = ____	I don't know 6 x 7 ...
5 x 7 = 35	... but I know my 5s
35 + 7 = 42	I have to add one more group of 7 to 35 and that equals 42

Break Apart Strategy

You are planting a garden in a 6’ by 8’ rectangle. You want to plant 2 types of vegetables. Find all the ways you can put a fence through the garden to make 2 smaller rectangles (one for each vegetable).



(5 x 8) + (1 x 8) =



(3 x 8) + (3 x 8) =

A Template to use when learning Derived Facts

Wow, that is a lot of strategies we just looked at!

What follows is a ***scaffold*** to help students think about using Foundational Facts to solve Derived Facts

How would you use Foundational Facts to solve this derived fact?

$$7 \times 8 =$$

Template for Solving Facts with Foundational Facts

X	1	2	10	5

Write one of the factors below the multiplication sign

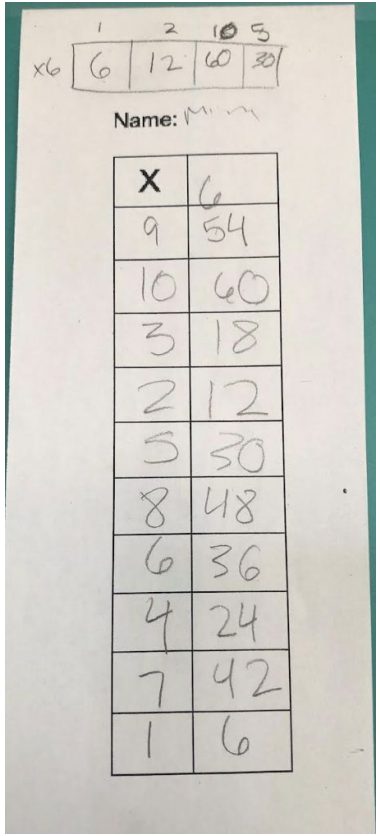
X	1	2	10	5
8				

Using the Foundational Facts, how would you solve 7 x 8 =

X	1	2	10	5
8	8	16	80	40

How would you use Foundational Facts to solve 17 x 8?

X	1	2	10	5
8	8	16	80	40



	1	2	10	5
x6	6	12	60	30

Name:

X	6
9	54
10	60
3	18
2	12
5	30
8	48
6	36
4	24
7	42
1	6

Partnering with Students on the Plan

The expectation is that fact fluency is continuous throughout the year. It is not a special unit and takes **time, effort and patience.**

Let students know:

- **that all this work on basic facts is to *help them move away from counting to automaticity***

Progress Monitoring

- Observation
- Interviews

We use these tools throughout the year in discussions with students to help them **monitor their own progress t**owards mastery.

Practicing the Facts

- Students need substantial and enjoyable Practice.
- Games can be used to achieve Mastery
- One way to use fact **games purposely** is to **incorporate strategy discussion** either before or after game play.

Play Heads Up (Salute)

1. In a group of 3, choose one person to be the leader.
2. Without looking at the cards, the 2 players choose a card and hold it to their foreheads facing outwards so the others can see them.
3. The leader says the sum/product of the 2 cards.
4. The other 2 players determine the value of the card on their forehead, based on hearing the sum/product and seeing the other card on the other player's forehead.
5. Both players share how they determined their numbers and must say the equation. *Can they say the inverse operation?* See Recording sheet
6. Discard cards and repeat.
7. The player to the right of the leader becomes the leader for the next round.

NOTE: Require the use of math language of factor, addend, product, sum

Math at Home: Make Practice Fun!

- Play games, especially games with dice and strategy games.
- Talk through strategies
- Focus on strategy selection, not speed. Speed will come with strategy practice.

Questions to Ask for Fact Fluency Games

- How did you solve that fact?
- Why did you choose that strategy?
- Are there other ways you could solve for that fact?
- What other facts might be solved with that strategy?
- When do you like to use that strategy (when is that strategy a good idea)?

Bedtime Math

Make Math Fun at home with Bedtime Math

Please feel free to contact me with any further questions.

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